Averaging pH vs. H⁺ Values

To the Editor:—In a recent letter to the editor, Giesecke¹ criticized statistical methods used by Stoetling² in reporting gastric-fluid pH changes following several preanesthetic medication regimens. Stoelting measured pH in gastric aspirates and derived mean and standard deviation values. Giesecke claimed that pH must first be converted to a real number, then statistically manipulated, and finally reconverted to pH form. Although details of the transformation were not given, it would appear Giesecke meant one should convert the pH to a derived hydrogen ion concentration ([H⁺]), average, take the negative logarithm, and call the result the average pH. He maintained that only a real number can be meaning and that pH, being a logarithm, is not real. (Parenthetically, a logarithmic transformation of a real number is most assuredly also a real number.) We believe that Giesecke is in error, and fear that acceptance of his letter by the editors of ANESTHESIOLOGY might reflect a new standard for the review of statistical procedures involving pH.

Both Stoelting and Giesecke seem to implicitly accept pH as the expression of gastric-fluid acidity. We agree with them. Although many have called for the abolition of pH notation and for the use instead of a derived [H⁺] in describing acidity,³⁻⁶ a consideration of thermodynamics applied to biologic systems confirms the superiority of pH over [H⁺] in relating acidity to physiologic function.⁶⁻⁷ Although pH was originally defined as pH = log 1/[H⁺], pH is now accepted as the measure of acidity without regard to that definition.⁷ pH is an independently determined variable; [H⁺] is a derived, dependent variable. Within certain tight constraints, it still remains true that pH = −log aH⁺, where aH⁺ = γ [H⁺]·aH₂O·hydrogen ion activity; γ: activity coefficient). It is likely that most physiologic processes affected by hydrogen ion respond in a manner proportional to the logarithm of the hydrogen ion activity.⁸

A series of pH measurements can be summarized by a sample mean and sample standard deviation. It is erroneous to take the antilog of the pH, invert, average, take the negative logarithm of the average, and call this number the mean pH.⁷ Let us consider a simple example. Given two samples of gastric fluid of equal volumes with pH 1 and 6, the mean pH is 3.5. When Giesecke’s method is used, the following calculations have to be made. First, the pH values are converted to [H⁺]; thus, pH 1 yields [H⁺] = 10⁻¹ mol/l and pH 6 gives [H⁺] = 10⁻⁶ mol/l. Next, the average of
In reply: — I have read with sustained fascination the correspondence by Pace et al. and others regarding mean pH as an expression of the central tendency of acidity in gastric specimens. Many arguments have been presented in favor of meaning pH values by adding them all together and dividing by "n" exactly the same as one would derive the mean of any other set of numbers. These arguments were so eloquent that I began to doubt my own conviction that this mathematical manipulation was not scientifically valid. My conviction was based largely on the knowledge that when one adds logarithms the antilogs that they represent are multiplied, not added. Further, when one divides a logarithm by a number "n", then one achieves the "n-th root" of the antilog which is represented. The controversy boils down to a simple question: "Which of the following is the best expression of the central tendency of acidity in a series of solutions of different pH?"

\[ \bar{x} = \frac{\sum x}{n} \]

or

\[ \bar{x} = \sqrt[n]{x \cdot x \cdot x \cdot \ldots} \]

I decided to test the question physically rather than just speculate on the theoretical mathematics. In the laboratory I added 100 ml of distilled water to each of five beakers. Using a continuously reading pH meter, I added hydrochloric acid or sodium hydroxide dropwise until the pH values of the five solutions read 2.045, 3.114, 4.131, 5.192 and 6.063. Triplicate observations and constant stirring were used to assure accuracy of the readings. To determine the central tendency of acidity of the solutions, I poured 25 ml of each of the five solutions together in a mixing flask and measured the pH of the resulting solution. If mean pH were a valid expression of the central tendency of acidity, then the pH of the resulting solution should read 4.109. As the actual reading was 2.758, which happens to be the pH of the mean hydrogen ion concentration in the resulting solution, I, therefore concluded that best expression of the central tendency of acidity in a series of solutions can be proven by...